The mathematics of electoral district allocation in New Zealand

Ian Westbrooke, Michael Ryan
Statistics New Zealand
Christchurch, New Zealand

Statistics NEW ZEALAND Te Tari Tatau

Published in October 2000 by
Statistics New Zealand
Te Tari Tatau
Wellington, New Zealand

Catalogue Number 01.095.0000
ISBN 0-478-20759-X

Contents

Contents - *continued*

List of tables

List of figures

Abstract

This paper outlines the implicit conceptual basis of New Zealand's electoral district allocation, as embodied in the Electoral Act 1993. We describe how these concepts are implemented in the Government Statistician's electoral calculations with examples from the 1997 calculations, and put the words of the Act into simple mathematical formulae. We show, using some more complex mathematics, how the Māori Option affects the number of Māori electoral districts, and describe an unexpected effect of the Option on the number of General electoral districts.

Introduction

In 1993 an overhaul of the New Zealand electoral system, with major changes in the representation of the Māori population, was embodied in the Electoral Act 1993. Since 1867, there has been separate representation for Māori in New Zealand's parliament, with separate Māori electoral districts. Between 1867 and 1993 the number of electoral districts was fixed at four. Initially this meant that a Māori member of parliament represented many more people than other members. By the turn of the century, with disease reducing the Māori population and with mass immigration into New Zealand, representation was approximately proportional to population and remained so around four decades. There was a significant reform in 1975, when Māori were given the choice between registering on the Māori or the General roll, with the introduction of a five-yearly Māori Electoral Option (Māori Option). By this stage, a Māori electoral district represented almost twice the population of a General electoral district (McRobie, 1989).

The changes in 1993 meant the number of Māori electoral districts could change, reflecting the proportion of Māori who chose the separate Māori roll. The electoral system became consistent in its approach for the Māori and General populations. The allocation of electoral districts to the two electoral populations Māori and General is now effectively based on four concepts (New Zealand Electoral Commission, 2000):

Concept 1 Electoral representation is based on total population resident in New Zealand, including children and others not eligible to vote, as determined by the most recent Census of Population and Dwellings held every five years. Therefore the main inputs come from the Census of Population and Dwellings, held every five years.

Concept 2 The total population is split into two segments:

- the Māori electoral population, which is a proportion of the population of Māori descent, the proportion depending on the percentage of all electors of Māori descent who choose the Māori roll
- the General electoral population, made up of the non-Māori population, plus the remainder of the Māori descent population, corresponding to the proportion of Māori electors who choose the General roll.

Concept 3 Approximately equal electoral populations in each electoral district, Māori or General.

Concept 4 A fixed number of General electoral districts in New Zealand's South Island, set at 16.

In section 1, we will show how these concepts are applied in the actual electoral calculations, using the 1997 calculations as the example. We end the section by expressing the electoral calculations in mathematical equations.

Section 2 discusses the Māori Option in detail. We show how changes during the Māori Option can affect the number of Māori and General electoral districts. In particular, we provide the basis for the technical information used in the 1997 Māori Option publicity.

Appendix 1 contains the more involved mathematical details for the Option calculations, while Appendix 2 reprints the technical information from the 1997 Māori Option publicity that was provided by Statistics New Zealand. Appendix 3 gives a glossary of variable names, and Appendix 4 provides a key definition from the Electoral Act 1993.

1. The Electoral Calculations

1.1 Māori and General electoral populations

As stated in Concept 2, the Māori electoral population (MEP) is a proportion of the population of Māori descent, the proportion depending on the percentage of all electors of Māori descent who choose the Māori roll. This percentage is of critical importance in all these calculations. We will refer to it as the **Māori ratio.** It is most clearly expressed in the formula

$$\mathbf{r} = \frac{\mathbf{m}}{\mathbf{m} + \mathbf{g}}$$

where

m is the number of persons on the Māori roll; and

g is the total number of Māori registered on the General roll.

The full details of the definition of MEP in the Electoral Act are given in Appendix 4.

The General electoral population (GEP) is the total population less the Māori electoral population. These calculations need to be carried out separately at national, island and electoral district level. (Nolan 1993).

To determine the number of electoral districts, the Electoral Act divides the population of New Zealand into three groups:

- the North Island General electoral population
- the South Island General electoral population
- the Māori electoral population (MEP)

Therefore, the Māori electoral population must be calculated at the national level (Table 1).

Table 1

Calculation of the 1997 Māori electoral population of New Zealand

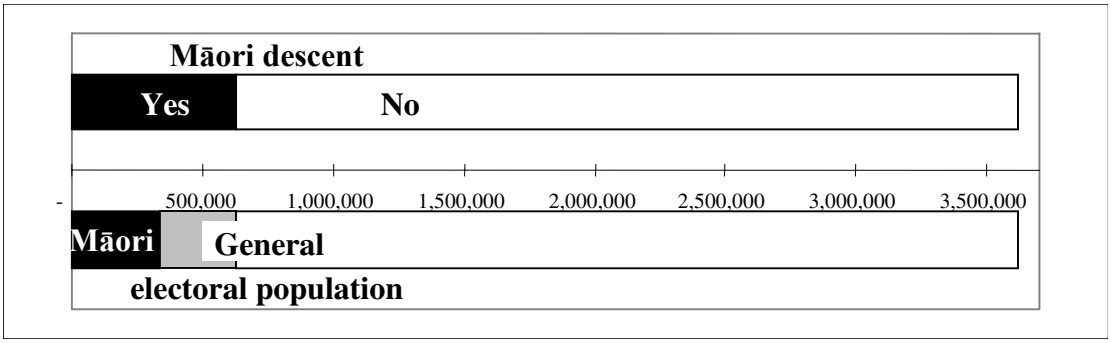
Population	Source	Number
New Zealand population (**u**)	1996 Census	3,618,302
Māori descent population (**d**)	1996 Census	628,429*
Māori on Māori roll (**m**)	Electoral rolls at end of 1997 Māori Option	163,310
Māori on General roll (**g**)	Electoral rolls at end of 1997 Māori Option	141,229
% of Māori choosing Māori roll (Māori ratio **r**)	$\mathbf{r} = \frac{\mathbf{m}}{\mathbf{m} + \mathbf{g}}$	53.6%
Māori electoral population	$\mathbf{MEP} = \mathbf{r}\,\mathbf{d}$	336,997

* The number includes an allocation of a proportion of those persons who answered 'don't know' or did not provide a response to the question on Māori descent. The allocation was based on standard statistical techniques (Westbrooke & Jones, 2000).

Figure 1 shows how the New Zealand population is divided between Māori and General electoral populations. The top bar shows the split of the New Zealand population by Māori descent. The bottom bar shows the subsequent split into Māori and General electoral populations. Part of the Māori descent population makes up the Māori electoral population. The remainder of the Māori descent population combines with the non-Māori population to give the General electoral population.

Figure 1

Māori and General electoral populations of New Zealand



We also need the General electoral populations for each island (Table 2).

Table 2

Calculation of General electoral population of North and South Islands

	North Island	South Island
Resident population (**u**)	2,718,188	899,384
Māori descent population (**d**)	544,562	83,416
% of Māori choosing Māori roll (**r**)	55.5%	41.0%
Māori electoral population	302,139	34,188
add Chatham Is GEP to SI GEP (Section 46 of Act)		480
General electoral population	= 2,718,188 - 302,139 =2,416,049	= 899,384 - 34,188+ 480 = 865,676

1.2 South Island quota

From concept 4, the number of General electoral districts in the South Island is fixed at 16. Therefore, dividing the South Island GEP by 16 gives the South Island quota, the average size of a South Island General electoral district.

For the 1997 calculations

$$\mathbf{q_s} = \frac{865{,}676}{16} = 54{,}105$$

1.3 Number of Māori electoral districts and Māori quota

Concept 3 states that electoral districts should have roughly the same electoral populations. Therefore the South Island quota is used to determine the number of electoral districts for the other populations.

The number of Māori electoral districts is determined by dividing the Māori electoral population by the South Island quota. In the 1997 calculations:

$$\mathbf{n} = \frac{336{,}997}{54{,}105}$$
$$= 6.23$$

This is rounded to the nearest integer, to give six Māori electoral districts in 1997.

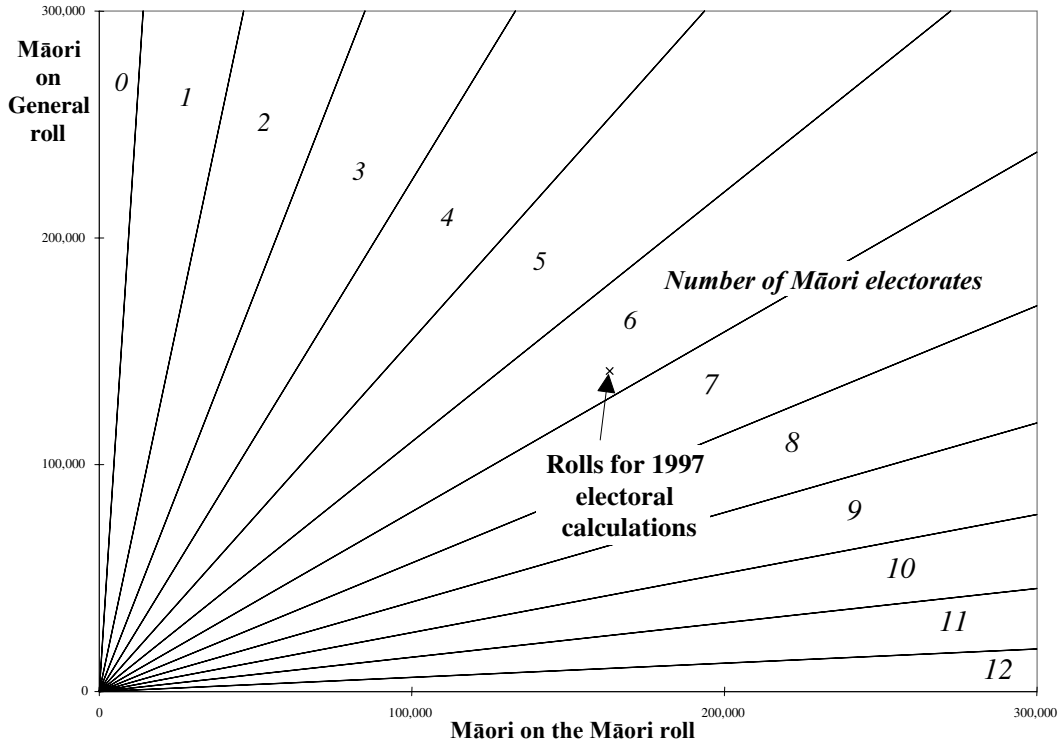
So the quota, or average size, for Māori electoral districts is then given by:

$$\mathbf{q_m} = \frac{336{,}997}{6}$$
$$= 56{,}166$$

With some assumptions (see Appendix 1) we can show how various states of the roll could lead to different numbers of Māori electoral districts. These are shown in Figure 2. This shows that if there were a high number of Māori on the general roll and few on the Māori roll, there would be few or even no Māori electoral districts. A higher enrolment on the Māori roll would lead to more Māori districts, up to a maximum of 12, based on the 1996 Census population. The state of the rolls at the end of the Māori Option, used in the 1997 electoral calculations, features in the centre of the figure.

Figure 2

Possible and actual number of Māori electoral districts, based on 1996 Census populations



1.4 North Island quota and number of electoral districts

The South Island quota is also used to determine the number of North Island General electoral districts, by dividing the island's GEP by the South Island quota:

In the 1997 calculations the number of North Island electoral districts is determined from:

$$\mathbf{h} = \frac{2{,}416{,}049}{54{,}105}$$
$$= 44.65$$

This is rounded to the nearest integer to give 45 North Island electoral districts. The quota, or average size, for North Island electoral districts is then given by:

$$\mathbf{q_n} = \frac{2{,}416{,}049}{45}$$
$$= 53{,}690$$

1.5 Approximately equal district populations

The way that the number of electoral districts is calculated ensures that the quota are as near to equal as possible, given that the number of electoral districts in each case has to be rounded to whole numbers.

The Electoral Act provides that district populations may vary by up to five percent from their quota.

Table 3

Summary of the 1997 electoral calculations

	Māori General	North Island General	South Island
Electoral population	336,997	2,416,049	865,676
Number of electoral districts	6	45	16
Quota	56,166	53,690	54,105
Minimum district population	53,358	51,006	51,400
Maximum district population	58,974	56,374	56,810

1.6 Setting new boundaries

Once the Government Statistician has carried out these calculations and reported them to the Surveyor General and other members of the Representation Commission, it is the task of the Surveyor General to draw up initial boundaries. The Commission then has six months to finalise new electoral boundaries.

1.7 Electoral equations

The calculations shown above can be put into general formulae, many of which will be needed below to assess the impact of changes in the Māori Option.

The Māori electoral population (MEP) for an area is given by the formula:

1. $$\mathbf{MEP} = \frac{\mathbf{m}}{\mathbf{m}+\mathbf{g}}\mathbf{d}$$

where

m is the number of persons on the Māori roll;

g is the total number of Māori registered on the General roll; and

d is the number of persons of Māori descent counted in the Census (Nolan 1993).

The same formula applies at all levels: New Zealand, North and South Islands, and electoral districts.

One detail that we will largely ignore is the special treatment of the Chatham Islands. Section 46 of the Act provides that its MEP is added to the North Island MEP, and the General electoral population (GEP) added to the South Island GEP. However this makes less than 0.1 percent difference in the South Island GEP. We have, however, included the Chathams in Table 2, to ensure that our results there coincide exactly with the official electoral calculations.

Using the Māori ratio $\mathbf{r} = \frac{\mathbf{m}}{\mathbf{m}+\mathbf{g}}$, equation **1**. becomes:

2. $$\mathbf{MEP} = \mathbf{r}\,\mathbf{d}$$

and the General electoral population (GEP) is given by:

3. $$\mathbf{GEP} = \mathbf{u} - \mathbf{r}\,\mathbf{d}$$

where **u** is the total usually resident population for the area of interest.

Dividing the South Island GEP by 16 gives the South Island quota ($\mathbf{q}_s$), the average size of a South Island General electoral district:

4. $$\mathbf{q}_s = \frac{\mathbf{u}_s - \mathbf{r}_s\mathbf{d}_s}{16}$$

where the subscript $_s$ refers to values for the South Island (that is $\mathbf{u}_s$, $\mathbf{r}_s$ and $\mathbf{d}_s$ are the same as for equation **1** or **3** but for the South Island only).

The ratio to determine the number of Māori electoral districts, **n**, is given by:

5.
$$\mathbf{n} = \frac{\mathbf{MEP_z}}{\mathbf{q_s}} = \frac{\mathbf{r_z\, d_z}}{\mathbf{q_s}}$$

where $\mathbf{q_s}$ is the South Island quota from above, and the subscript $_z$ refers to values for the whole of New Zealand.

Figure 2 shows that the maximum possible number of Māori electoral districts based on the 1996 populations was 12. The maximum value of **n** can be calculated directly, as it would result from all Māori enrolling on the Māori roll, that is putting $\mathbf{r_s}$ =1 in **4**, and $\mathbf{r_z}$=1 in **5**.

Let **h** be the ratio determining the number of General electoral districts in the North Island. Using the same notation as previously, with subscript $_n$ to refer to the North Island:

6.
$$\mathbf{h} = \frac{\mathbf{GEP_n}}{\mathbf{q_s}}$$

2. Calculating The Impact Of The Māori Option

As mentioned above, the 1993 Electoral Act made an important change–the number of Māori electoral districts became dependent on the choice of roll by Māori. Before electoral boundaries are redrawn each five years, there is a four-month period during which Māori may change from one roll to the other, a process called the Māori Option. At each Option, registered Māori electors may transfer from either the Māori roll or the General roll to the other roll. Māori who have passed 18 years of age and are not yet registered may register at any time as electors on either roll.

Previously the choices by Māori made no difference to the number of Māori electoral districts, which meant political interest in the Māori Option was limited. From 1993 the Māori Option acquired greater importance to Māori and to the New Zealand political process. The Report of the Royal Commission on the Electoral System (1986) predicted

"Indeed, if the number of Māori seats were to be determined on the basis of the number opting to go on the Māori roll, the option would become much more important in terms of the mana of the Māori people...." (p107).

The first Māori Option under the new law was held in 1994. The new situation created substantial interest and also stimulated an increase in Māori enrolment overall. During the period leading up to the Option, Statistics New Zealand received a number of requests for assessments of the effects of possible changes in the rolls. Enquirers wanted to know how the Māori Option might affect the number of Māori electoral districts. Based on our experience in 1994, we found that two central questions were:

- How many Māori need to move from one roll to the other to increase (or decrease) the number of Māori electoral districts by one.
- How many new registrations on the Māori roll are required to increase the number of Māori electoral districts by one.

A further related question is:

- How do changes in the Māori Option affect the number of General electoral districts.

The answers to such questions are not readily apparent from the 1993 Act. In this part of the paper, we express the relevant definitions of the Act in mathematical formulae and show how such questions can be answered.

2.1 Processes in the Māori Option

During the Māori Option three main processes can affect Māori enrolments:

- Transfers from one roll to the other. We will focus on transfers from the General roll to the Māori roll and these will be taken as positive. Transfers in the other direction, from the Māori roll to the General roll, will be represented by negative values.
- New enrolments by Māori on the Māori roll.
- New enrolments by Māori on the General roll.

Transfers may only take place during the Option period, while new enrolments are allowed at any time. The rolls also have some loss, generally small, due to factors such as deaths or migration.

The net effect of these three processes determines the number of Māori on each roll at the end of a Māori Option; and determines the Māori ratios which affect the Māori electoral population. Although we can distinguish the above three processes, only the relative sizes of Māori enrolments on the two rolls affect the electoral calculations. Therefore, changes in the numbers of Māori on the Māori roll and on the General roll can be expressed in terms of just two of the above three processes.

In both the 1994 and 1997 Māori Options there were net increases in the Māori roll due to both transfers and new enrolments. Therefore we focus on the first two processes as this choice is likely to be of general interest. The third process, new enrolments by Māori on the General roll, can be represented by the same number of new enrolments on the Māori roll together with the same number of transfers from the Māori roll to the General roll.

To clarify how the Māori Option might affect the number of Māori seats we need to relate the three option processes to required changes in the Māori ratios to bring about specified increases in the number of Māori electoral districts. Two useful simple cases are described next.

Note: Appendix 3, on page 22, provides a list of all the variables used in the following formulae.

2.1.1 Transfers alone:

For transfers, lets say the number of Māori on the Māori roll **m** increases by **x**, while the total of Māori on both rolls (**t=m+g**) remains constant, so:

7. $\mathbf{x = \Delta rt}$

where **Δ** indicates the change in a variable.

2.1.2 New enrolments alone:

Now consider new enrolments of Māori on the Māori roll, formerly not on any roll, or not identified as Māori. In general, for new enrolments, both **m** and **t** increase by **y**. So:

$$\mathbf{8.} \qquad \Delta \mathbf{r} = \frac{\mathbf{m}+\mathbf{y}}{\mathbf{t}+\mathbf{y}} - \frac{\mathbf{m}}{\mathbf{t}}$$

It follows, using **m=rt**, that

$$\mathbf{9.} \qquad \mathbf{y} = \frac{1}{1-\mathbf{r}-\Delta\mathbf{r}}\Delta\mathbf{r}\,\mathbf{t}$$

So we can see by comparing **8** and **9** that it requires more new enrolments than transfers to achieve the same change in **r**, by a factor of $\frac{1}{1-\mathbf{r}-\Delta\mathbf{r}}$. This same factor holds for the relative impact on **n**, the number of Māori electoral districts. Currently values of **r** are about 0.5, so transfers have around twice the impact of new enrolments.

2.1.3 Transfers and new enrolments

In practice, both transfers and new enrolments will take place in a Māori Option. Using the same notation, we can express both effects:

$$\Delta \mathbf{r} = \frac{\mathbf{m}+\mathbf{x}+\mathbf{y}}{\mathbf{t}+\mathbf{y}} - \mathbf{r}$$

and it follows, again using **m=rt**, that:

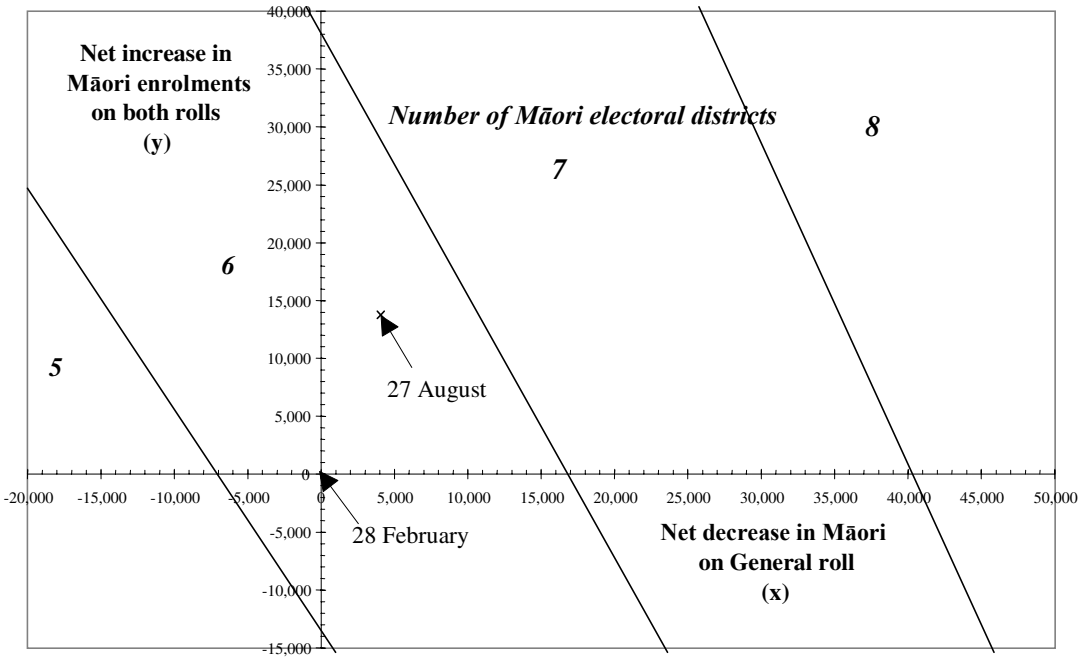
$$\mathbf{10.} \qquad \mathbf{x} + (1-\mathbf{r}-\Delta\mathbf{r})\mathbf{y} = \Delta\mathbf{r}\,\mathbf{t}$$

At the start of the Māori Option, **r** and **t** are fixed. There are various combinations of **x** and **y** that can result in a particular change in **Δr**. For a given level of **Δr**, **10** defines a series of straight lines in the **xy** plane. The lines are not parallel, as the slope varies with **Δr**. By deriving the levels of **Δr** which correspond to the changes in **n** to gain or lose additional Māori electoral districts, as we do below, we can define the boundaries for regions of **x** and **y** which correspond to the different possible numbers of Māori electoral districts, as shown in Figure 3. (Figure 3 uses a slightly different definition of **x** and **y**. The two definitions are equivalent if no other processes are affecting the rolls, that is no new Māori enrolments on the General roll, and no losses due to deaths, etc.)

The advantage of Figure 3 over Figure 2 is that it shows the effects on the number of Māori electoral districts of *changes* in the electoral rolls likely during the Māori Option. It shows them relative to a reference state of the roll, and is especially appropriate for seeing the effects of relatively small changes in the roll. Figure 3 shows clearly the impact of the changes in the roll during the 1997 Option. Figure 2 is better for seeing the effect very different states of the roll would have.

Figure 3

Māori enrolment in 1997 Māori Option and its effect on the number of Māori electoral districts



Note: The location of a point in the diagram, defined by the net decrease in Māori on the General roll and net increase in Māori enrolments on both rolls, determines the number of Māori electoral districts.

2.2 Impact of Māori Option on number of Māori electoral districts

Combining equations **2**, **4** and **5** gives:

$$\mathbf{11.} \quad \mathbf{n} = \frac{\mathbf{r_z d_z}}{\mathbf{q_s}} = \mathbf{r_z} \frac{16\mathbf{d_z}}{\mathbf{u_s} - \mathbf{r_s d_s}}$$

Equation **11** shows that the number of Māori electoral districts is affected by changes in the Māori ratio during the Māori Option for both the whole of New Zealand ($\mathbf{r_z}$) and for just the South Island ($\mathbf{r_s}$). The other parts of equation **11** are not affected by the Māori Option. To assess how **n** changes in response to the patterns of Māori registration we must consider the changes in the Māori ratio in both the South Island and New Zealand as a whole.

2.2.1 Simplified approach - taking the South Island Māori ratio as fixed

The census Māori descent population $\mathbf{d_z}$ is unaffected by the Māori Option, and if the South Island Māori ratio is held constant, then $\mathbf{q_s}$ is also constant. Treating $\mathbf{d_z}$ and $\mathbf{q_s}$ as constants implies:

12. $$\Delta \mathbf{n} = \Delta \mathbf{r_z} \frac{\mathbf{d_z}}{\mathbf{q_s}}$$

where Δ is used to denote a change in a variable.

Equation **12** can be used to estimate what change in the New Zealand Māori ratio is required to bring about a specified change in the number of Māori electoral districts. For some practical purposes, this simple approach will be adequate. For example, it gives reasonable estimates of the number of transfers or new enrolments required to gain or lose one Māori electoral district. However, we found that the assumption of a fixed South Island Māori ratio was too simple for wider analysis. It gives misleading results for the number of Māori electoral districts that would occur if all Māori chose the Māori roll. It also suggests that there is a simple trade off between Māori and North Island General electoral districts which is not true.

2.2.2 Relaxing the assumption of fixed South Island quota, q_s

For the calculations used in the 1997 Māori Option publicity and in Figure 3, we replaced the assumption of a constant South Island quota (which is the same as assuming that the Māori ratio is constant in the South Island). The replacement assumption was that the Māori ratio moves by the same amount in South Island as in New Zealand as a whole, but from a different starting point. This assumption is consistent with the limited data available. In 1994, the ratio in the South Island moved from 0.30 to 0.39, while that in the North Island moved from 0.41 to 0.52. The 1997 Option results were also consistent with this assumption, with both ratios moving by about 0.04. Allowing the South Island Māori ratio to vary leads to substantially more complex formulae, and makes a noticeable difference in the estimates of the changes in the roll needed to change the number of Māori electoral districts. The details of the mathematics involved are given in Appendix 1.

For Figure 2, we used a more complex assumption: that the South Island Māori ratio is a quadratic function of the New Zealand ratio. This produces good approximations for the number of electoral districts for a given state of the rolls nationally, including at the very extremes. The exact form of the quadratic used will need to be reviewed for future applications, and changed if it is no longer consistent with the state of the rolls. The simpler assumption, which held the South Island ratio fixed or set it as a fixed proportion of the New Zealand ratio, distorted the graph, in particular giving unacceptably low values for the maximum number of Māori electoral districts. The formulae behind Figure 2 are also given in Appendix 1.

No special assumptions were needed to derive the effects of the Māori Option on the number of General electoral districts, described below.

2.3 The 1997 Māori Option

To illustrate the implementation of our analysis of the impact of the Māori Option, we show, in Tables 4, 5 and 6, the calculations made to provide information for the official Māori Option publicity. Using the state of the rolls at 28 February 1997, we compare the results of our assessments with the definitive electoral calculations. The numbers are less than a thousand lower than the figures at the beginning of the Option on 28 April, and the differences make no significant difference to the analysis.

Table 4

Populations and enrolments

	Symbol	South Island ($_s$)	New Zealand ($_z$)
Enrolments as at 28 February			
Māori on the Māori roll	$\mathbf{m}$	14,188	145,473
Māori on the General roll	$\mathbf{g}$	23,815	145,297
Māori ratio	$\mathbf{r}$	37.3%	50.0%
1996 Census ordinarily resident populations (5 March)			
Māori descent population (including an allocation of those who answered 'Don't know' or failed to specify)	$\mathbf{d}$	83,416	628,429
Total	$\mathbf{u}$	899,384	3,618,302

Table 5

Electoral calculations if based on state of Māori rolls before 1997 Māori Option (28 February)

	Formula	Calculated value
South Island quota	$\mathbf{q}_s = \frac{\mathbf{u}_s - \mathbf{d}_s\mathbf{r}_s}{16}$	54,265
Ratio to determine number of Māori electoral districts	$\mathbf{n} = \frac{\mathbf{d}_z}{\mathbf{q}_s}\mathbf{r}$	5.79
Increase in $\mathbf{n}$ to gain 7th Māori electoral district	$6.5-\mathbf{n}$	0.71
Decrease in $\mathbf{n}$ to lose 6th Māori electoral district	$\mathbf{n}-5.5$	0.29

Table 6

Estimated transfers and new enrolments to gain or lose Māori electoral districts

	Number of Māori electoral districts	Simplified assumption: q constant (α=0)	Preferred assumption: SI and NZ ratio move together (α=0)
Transfers (x)			
	7	17,730	16,823
	8	42,838	40,329
	5	-7,378*	-7,056*
New enrolments on the Māori roll (y)			
	7	40,412	38,074
	8	121,570	111,713

* Negative transfers refer to movements from the Māori to the General roll

Based on these calculations, Statistics New Zealand prepared technical information for the main Māori Option publicity. This was posted to all electors who identified themselves as Māori when they enrolled, and distributed widely as part of the Option campaign. The technical information is reprinted in Appendix 2.

How well did the calculations compare with the results of the final electoral calculation? From Table 5, we see that the ratio determining the number of Māori electoral districts, **n**, stood at 5.79 at 28 February 1997. As noted above, there are three processes taking place in the Option, transfers between rolls, and new enrolments on either roll. In addition, there will be a small loss from the rolls due to deaths, migration and other effects. For the purposes of estimating the impact on the number of Māori electoral districts, these processes can be summarised into two numbers: the net increase in the total number of Māori enrolled (which can be regarded as net new enrolments), and the net loss of Māori from the General roll (which can be regarded as the net effect of transfers from the General to the Māori roll allowing for the effect of new enrolments on the General roll). The net increase in Māori on both rolls from 28 February to the end of the Option was 13,769, while the net loss of Māori on the General roll was 4,068.

Based on the estimates in Table 6, the expected change in **n** during the Māori Option is 0.40 using the simple assumption (α=0 ie q constant), and 0.43 for our preferred assumption that the South Island and New Zealand Māori ratio move together (α=1). The actual change was 0.44, giving the final value that **n** is 6.23, rounding to six electoral districts. We see that the second assumption gives a better estimate, and that the calculations used for the technical information in the Option provided an accurate picture for Māori electors of the impact of choosing one roll or the other.

2.4 How the Māori Option affects the number of General electoral districts

It is widely assumed that there is a simple trade-off between the number of Māori electoral districts and the number of General electoral districts. This is not true.

The number of South Island General electoral districts is fixed at 16. The results of the Māori Option affect the determination of the South Island Māori electoral population, and this in turn affects the South Island quota. The South Island quota affects the calculation of both the number of Māori electoral districts and the number of North Island General electoral districts. There is a simple trade off between Māori and General electoral districts only if the Māori ratio in the South Island remains fixed, which would seem unlikely if the Māori ratio had changed in the North Island.

If the Māori ratio increases by similar amounts in both the North and South Islands, we show in Appendix 1 that the reduction in the number of North Island General electoral districts is almost halved by the effect on the South Island quota.

Combining this effect with the overall increases in population between 1991 to 1996 means that the new number of North Island General electoral districts could not have dropped below 43, even if the number of Māori electoral districts increased to the maximum possible, 12. This compares with the 44 North Island General electoral districts in the current Parliament.

The overall number of electoral district seats and list seats in Parliament is fixed at 120 in most situations. Therefore as the number of Māori and General electoral districts increases in response to growing population and changes in the Māori Option, this will generally be offset by fewer list seats.

3. Conclusion

This paper has shown that the calculations required under the 1993 Electoral Act for the allocation of electoral districts can be presented in terms of simple formulae which make the whole process clearer. From these formulae, it is possible to show in simple terms how changes during the Māori Option affect the allocation of electoral districts. This information can help all interested citizens understand the electoral allocation process. In particular, it can assist those making choices in the Māori Option.

4. References

Frank Nolan (1993), Some electoral mathematics, *The New Zealand Statistician*, 28(1), pp.11-18.

McRobie, A. (1989), *New Zealand Electoral Atlas*, Government Printing Office: Wellington.

New Zealand Electoral Commission (2000), *The New Zealand Electoral Companion, 2nd Edition*, NZ Electoral Commission: Wellington.

Royal Commission on the Electoral System (1986), *Towards a Better Democracy*, Government Printer: Wellington.

Westbrooke, I.M. & Jones, L.K. (2000), *Imputation of Māori Descent for Electoral Calculations*, Statistics New Zealand Research Report.

Appendix 1: Mathematical Details

1. Effects of Māori Option on Number of Māori Electoral Districts

From equation **5**.

$$\mathbf{n} = \frac{\mathbf{r_z d_z}}{\mathbf{q_s}}$$

$$\mathbf{13.} \quad = \frac{16\mathbf{r_z d_z}}{\mathbf{u_s} - \mathbf{r_s d_s}}$$

Only $\mathbf{r_s}$ and $\mathbf{r_z}$ change in the Māori Option.

Let them change by $\Delta\mathbf{r_s}$ and $\Delta\mathbf{r_z}$ respectively.

Then the change in **n**, $\Delta\mathbf{n}$ is given by:

$$\mathbf{14.} \quad \Delta\mathbf{n} = \frac{16(\mathbf{r_z} + \Delta\mathbf{r_z})\mathbf{d_z}}{\mathbf{u_s} - (\mathbf{r_s} + \Delta\mathbf{r_s})\mathbf{d_s}} - \frac{16\mathbf{r_z d_z}}{\mathbf{u_s} - \mathbf{r_s d_s}}$$

This reduces to

$$\mathbf{15.} \quad \Delta\mathbf{n} = 16\mathbf{d_z u_s} \frac{\Delta\mathbf{r_z}(\mathbf{u_s} - \mathbf{r_s d_s}) + \Delta\mathbf{r_s r_z d_s}}{\{\mathbf{u_s} - (\mathbf{r_s} + \Delta\mathbf{r_s})\mathbf{d_s}\}\{\mathbf{u_s} - \mathbf{r_s d_s}\}}$$

Given $\Delta\mathbf{n}$ there is no unique solution for $\Delta\mathbf{r_s}$ and $\Delta\mathbf{r_z}$.

It is useful to transform variables by taking $\Delta\mathbf{r_s}=\alpha\,\Delta\mathbf{r_z}$. In general *a* is a variable, but we can choose different values to implement different assumptions.

We can now drop the subscript on $\Delta\mathbf{r}$, giving

$$\mathbf{16.} \quad \Delta\mathbf{n} = 16\mathbf{d_z u_s} \frac{\Delta\mathbf{r}(\mathbf{u_s} - \mathbf{d_s r_s}) + \alpha\Delta\mathbf{r r_z d_s}}{\{\mathbf{u_s} - \mathbf{d_s}(\mathbf{r_s} + \alpha\Delta\mathbf{r})\}\{\mathbf{u_s} - \mathbf{d_s r_s}\}}$$

From this equation we derive

$$\mathbf{17.} \quad \Delta\mathbf{r} = \frac{\mathbf{r_z}\Delta\mathbf{n}}{\mathbf{n}} \frac{1}{1 + \alpha\,\mathbf{r_z}\dfrac{\mathbf{d_s}}{\mathbf{u_s} - \mathbf{d_s r_s}} + \alpha\,\Delta\mathbf{n}\dfrac{\mathbf{d_s}}{16\mathbf{d_z}}}$$

Based on this, and equations **7** or **9**, it is straightforward to derive the numbers of transfers or new enrolments required to reach a particular number of Māori electoral districts, based on knowledge of census population and the current state of Māori enrolments, plus an assumption about the value of α. If we take α=0 (ie assume that the South Island Māori ratio does not change), then we get back to equation **12.**

2. Calculating the Number of Māori Electoral Districts for Any Level of Māori Enrolments

To derive the number of Māori electoral districts for any state of Māori enrolments shown in Figure 2, we need an assumption about the relationship of $\mathbf{r}_s$ and $\mathbf{r}_z$.

In the extreme cases, if all Māori chose the Māori roll, then $\mathbf{r}_s$=1 and $\mathbf{r}_z$=1; and if all chose the General roll then $\mathbf{r}_s$=0 and $\mathbf{r}_z$=0. We want the relationship to give these results and also a good approximation to the current values of $\mathbf{r}_s$ and $\mathbf{r}_z$. A quadratic relationship can achieve this. Taking:

18. $$\mathbf{r_s} = \frac{\mathbf{r_z}(\mathbf{r_z}+1)}{2}$$

gives the desired values at the extremes, and when $\mathbf{r}_z$=0.5 then $\mathbf{r}_s$=0.375, very close to the values in early 1997.

Substituting in equation **11** gives:

19. $$\mathbf{n} = \mathbf{r_z}\frac{16\mathbf{d_z}}{\mathbf{u_s} - \frac{1}{2}\mathbf{r_z}(\mathbf{r_z}+1)\mathbf{d_s}}$$

This gives a quadratic in $\mathbf{r}_z$

20. $$\mathbf{n d_s r_z^2} + (\mathbf{n d_s} + 32\mathbf{d_z})\mathbf{r_z} - 2\mathbf{n u_s} = 0$$

Taking the positive root for $\mathbf{r}_z$ gives

21. $$\mathbf{r_z} = \frac{\sqrt{(\mathbf{n d_s} + 32\mathbf{d_z})^2 + 8\mathbf{n^2 d_s u_s}} - (\mathbf{n d_s} + 32\mathbf{d_z})}{2\mathbf{n d_s}}$$

The lines and regions shown in Figure 2 were derived from this equation.

3. Effects of Changes in the Māori Roll on the Number of General Electoral Districts in the North Island

Let **h** be the ratio determining the number of General electoral districts in the North Island. Using the previous notation, with subscript $_n$ to refer to the North Island:

22. $$\mathbf{h} = \frac{\mathbf{GEP_n}}{\mathbf{q_s}} = 16\frac{\mathbf{u_n} - \mathbf{r_n d_n}}{\mathbf{u_s} - \mathbf{r_s d_s}}$$

Let $\mathbf{r}_n$ change by $\Delta\mathbf{r}_n$ and $\mathbf{r}_s$ change by $\Delta\mathbf{r}_s$.

23. Then: $$\Delta\mathbf{h} = 16\frac{\mathbf{u_n} - (\mathbf{r_n} + \Delta\mathbf{r_n})\mathbf{d_n}}{\mathbf{u_s} - (\mathbf{r_s} + \Delta\mathbf{r_s})\mathbf{d_s}} - 16\frac{\mathbf{u_n} - \mathbf{r_n d_n}}{\mathbf{u_s} - \mathbf{r_s d_s}}$$

If $\Delta\mathbf{r}_s$=0, then

24. $$\Delta\mathbf{h} = -16\frac{\Delta\mathbf{r_n d_n}}{\mathbf{u_s} - \mathbf{r_s d_s}}$$

which is the same as the increase in the number of Māori electoral districts. That is, there is a simple trade-off between Māori and North Island General electoral districts if the South Island Māori ratio remains the same.

In the more general case:

25. $$\Delta\mathbf{h} = -16\frac{\mathbf{d}_n}{\mathbf{u}_s - (\mathbf{r}_s + \Delta\mathbf{r}_s)\mathbf{d}_s}\Delta\mathbf{r}_n + 16\frac{(\mathbf{u}_n - \mathbf{r}_n\mathbf{d}_n)\mathbf{d}_s}{(\mathbf{u}_s - (\mathbf{r}_s + \Delta\mathbf{r}_s)\mathbf{d}_s)(\mathbf{u}_s - \mathbf{r}_s\mathbf{d}_s)}\Delta\mathbf{r}_s$$

The trade-off effect is essentially given by the first term, and the effect of the change in the South Island quota is given by the second term.

The approximate effect of $\Delta\mathbf{r}_n$ and $\Delta\mathbf{r}_s$ values can be assessed by putting $\mathbf{r}_s + \Delta\mathbf{r}_s$ into this formula. Using the data in Table 4, this gives, approximately, for the 1996 Census and electoral rolls in February 1997:

26. $$\Delta\mathbf{h} \cong -10\Delta\mathbf{r}_n + 4.3\Delta\mathbf{r}_s$$

Thus if there is the same change in the Māori ratio in both islands, the effect on the South Island quota offsets almost a half of the loss of North Island General electoral districts.

Appendix 2: Technical Information From Māori Option Publicity

Technical Information

(Supplied by Statistics New Zealand)

The choice of every Māori elector whether to go on the Māori or General Roll will help determine the number of Māori electorates in the future.

The roll you choose helps decide how many Māori electorates there will be when boundaries are redrawn.

- More Māori electors enrolled on the Māori Roll can mean more Māori electorates.
- More Māori electors enrolled on the General Roll can mean fewer Māori electorates.

There are 5 Māori electorates in the current Parliament, based on the outcome of the 1994 Māori Option. Based on 28 February 1997 enrolments and provisional 1996 population census information, there would be 6 Māori electorates if new boundaries were drawn at that date.

If you are currently on the General Roll and stated that you are Māori, then if you and about 17,000 others transfer to the Māori Roll, the number of Māori electorates would increase to 7. A further 24,000 transfers would increase the number of Māori electorates to 8.

If you are currently on the Māori Roll, then if you and about 7,000 others transfer to the General roll, the number of Māori electorates would revert back to the existing level of 5.

New enrolments by people of Māori descent during the Māori Option also affect the final number of Māori electorates but not as much as changes from one roll to the other. The final outcome will depend on the combination of all the transfers and new enrolments, including your decision.

One example: a combination of 11,000 transfers from the General to the Māori Roll, and 15,000 new enrolments on the Māori Roll, would increase the number of Māori electorates to 7.

Transfers from the Māori to the General Roll and new enrolments of Māori on the General Roll may reduce the number of Māori electorates.

Please note the numbers in the examples shown here are simplified for easy reference.

Fuller details are available from the Electoral Enrolment Centre.

Appendix 3:
Glossary Of Variable Names

Variables	Description	Source of data variables
d	Māori descent population	from census
u	usually or ordinarily resident population	from census
m	number of Māori on Māori roll	from electoral enrolments
g	number of Māori on the General roll	from electoral enrolments
t	number of Māori on both rolls (=**m**+**t**)	from electoral enrolments
r	Māori ratio $= \frac{\mathbf{m}}{\mathbf{t}}$	
q	electoral quota (average size of electoral population in each electoral districts for that population–Māori, North Island General and South Island General)	
n	ratio determining the number of Māori electoral districts	
x	Transfers from General to Māori roll	
y	New enrolments on the Māori roll	
a	factor for relationship between changes in the New Zealand and South Island Māori ratio	
Δ	indicates a change in a variable	
SUBSCRIPTS		
$_{n}$	North Island	
$_{s}$	South Island	
$_{z}$	New Zealand	

Appendix 4: Definition Of Māori Electoral Population

The Electoral Act 1993, Section 3. Interpretation, states that:

> "Māori electoral population" means a figure representing both the persons registered as electors of the Māori electoral districts and a proportion of the persons of New Zealand Māori descent who are not registered as electors of any electoral district and a proportion of the persons of New Zealand Māori descent under the age of 18 years, which figure shall be fixed—
>
> (a) By ascertaining a proportion (which shall be determined by dividing the total number of persons registered, as at the close of the last day of the period specified in the last notice published under section 77 (2) of this Act, as electors of Māori electoral districts by the number of persons of New Zealand Māori descent registered, as at the close of that day, as electors of either General electoral districts or Māori electoral districts); and
>
> (b) By applying the proportion ascertained under paragraph (a) of this definition to the total number of ordinarily resident persons of New Zealand Māori descent as determined by the last periodical census: